Chapter 17

Design of Experiment (DoE)

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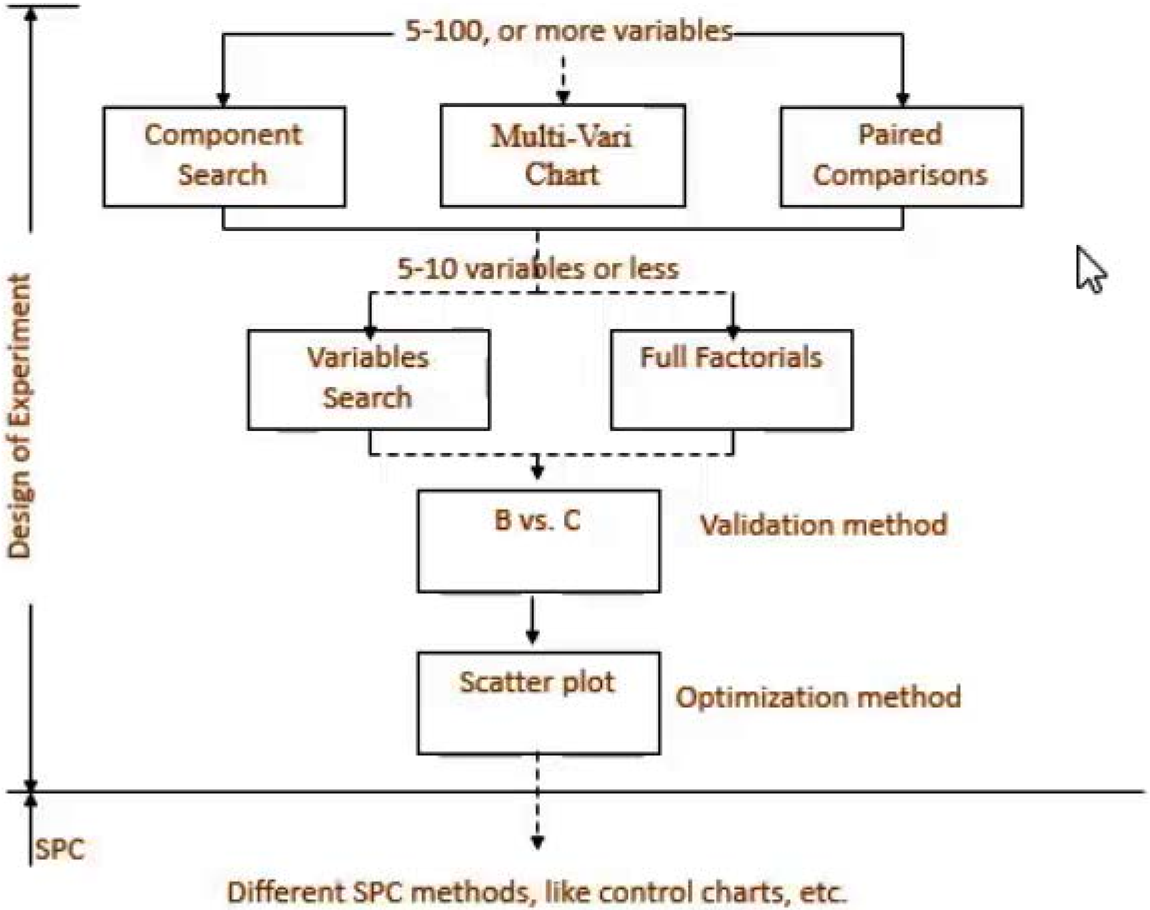
17.1 Introduction to DoE

Design of Experiment (DOE) is systematic and highly structured method for identifying important variables, or quality factors that affect quality, and ultimately eliminating those preferably at the design stage before it goes to the customer.

1. Identify the important variables or parameters that affect quality.
2. Perform DoE experimentations.
3. Eliminate Red X (most serious, or that factor which has serious effect on quality) and Pink X (moderately serious, which has moderate impact on quality) problems.
4. Opening up the tolerances on the unimportant variables can help in reducing costs.

Reducing number of variables from many to a few for control purpose is known as "Homing in" technique.

Figure 17.1: DOE methods, commonly used in industries.



17.4 Introduction to Full Factorial Analysis

Variable Search Homing-in Technique → Can consider five or more factors at a time

Full Factorial Homing-in technique → Can consider four or fewer factors at a time because of complexity in computation

Two factors and two levels (current level, improved level) of each factor ⟶ 2^2 factorial.

Three factors with two levels each ⟶ 2^3 factorial,

Four factors with two levels each ⟶ 2^4 factorial.

A 2^4 factorial analysis involves 16 (i.e. $2 \times 2 \times 2 \times 2$) groups of tests, or combinations.

One group of test means experiments with a particular combination of factors. For instance, if A, B, C and D are the four factors having two levels each (– and +), then a combination of factors may be A+, B-, C- and D+. One group of test is termed as '**Cell**' here in this chapter.

Quality experts are of the view that consideration of four or fewer factors is sufficient.

Thus, Full Factorial technique has become a powerful and popular homing-in technique, and hence used frequently in practice.

17.4.2 Methodology

The complete Full Factorial analysis involves two main steps:

1. Preparation of a combinatorial matrix, which identifies the impacts of all individual factors having two levels for each factor.

2. Preparation of an ANOVA (Analysis of Variance) table to identify main effect and interaction effect, and possibly Red X factors and Pink X factors.

An Example

A medicated chocolate product is produced for throat inflammation. Since this chocolate is supposed to be chewed for a long time, its 'hardness and fragility' is an important quality factor. If hardness of the chocolates is too high, it becomes too fragile, and difficult to chew. Inspection shows that currently 10% of the produced chocolates are rejected because they are produced broken. This happens because of excessive hardness. If hardness is too low, it becomes soft, what the consumers don't like in this particular type of chocolate as chewing becomes irritating because of sticky nature.

Table17.2: Two levels of four factors.

Factors	Levels (– or +)	
	Current value (–)	Experimental (+)
(A). Moister content (%)	10	15
(B) Mixing temperature (degree Celsius)	100	90
(C) Mixing time (minutes)	10	15
(D) Rotational speed of mixer chamber (rpm)	50	70

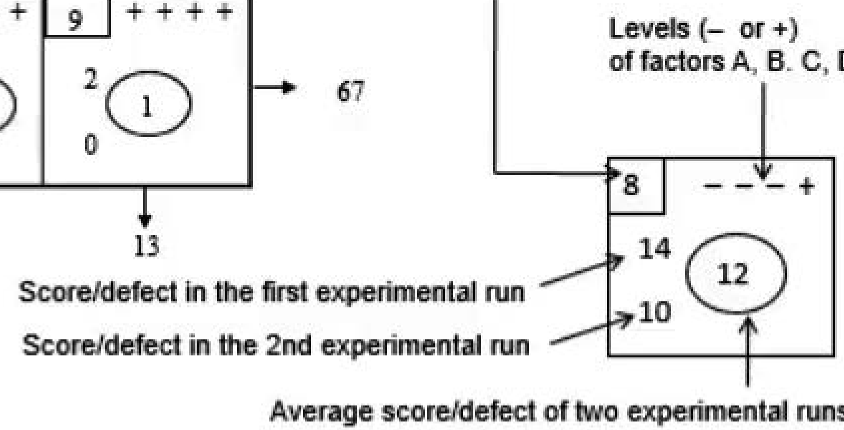
Table 17.3: Combinatorial matrix

Factors A and B

		A–		A+		
		B–	B+	B–	B+	
C–	D–	[1] − − − − 19 / 15 (17)	[3] − + − − 15 / 9 (12)	[7] + − − − 110 / 120 (115)	[12] + + − − 5 / 5 (5)	→ 149
C–	D+	[8] − − − + 14 / 10 (12)	[10] − + − + 57 / 53 (55)	[4] + − − + 10 / 4 (7)	[13] + + − + 3 / 3 (3)	→ 77
C+	D–	[6] − − + − 11 / 5 (8)	[2] − + + − 54 / 46 (50)	[11] + − + − 42 / 44 (43)	[15] + + + − 3 / 5 (4)	→ 105
C+	D+	[14] − − + + 40 / 44 (42)	[16] − + + + 15 / 21 (18)	[5] + − + + 6 / 6 (6)	[9] + + + + 2 / 0 (1)	→ 67
		↓ 79	↓ 135	↓ 171	↓ 13	

Legend

Random number in the sequence of experimental runs, e.g. in this case, A–, B–, C– and D+ combination was executed in the 8th position in the sequence of 16 experiments.



From the combinatorial matrix (Table 17.3), total individual scores/defects are calculated as in Table 17.4.

Table 17.4: Contributions of individual factors

Factors	Total scores/readings/defects	Difference
A–	79 + 135 = 214	A– is worse than A+ by 30 defects
A+	171 + 13 = 184	
B–	79 + 171 = 250	B– is worse than B+ by 102 defects
B+	135 + 13 = 148	
C–	149 + 77 = 226	C– is worse than C+ by 54 defects
C+	105 + 67 = 172	
D–	149 + 105 = 254	D– is worse than D+ by 110 defects
D+	77 + 67 = 144	

Here, the difference between A– and A+ (it is 30 in this case) is due to factor A alone.

The same explanations apply to other factors as well.

In this particular example, results show that level plus (+) of all four factors perform better than level minus (–) which are the current levels.

An ANOVA table can be prepared based on the above results, as shown in Table 17.5 (next slide).

Table 17.5: Full Factorial ANOVA table.

Cell group	Factors				2 factors interactions						3 factors interactions				4 factors interactions	Output
	A	B	C	D	AB	AC	BC	AD	BD	CD	ABC	ABD	ACD	BCD	ABCD	
1	−	−	−	−	+	+	+	+	+	+	−	−	−	−	+	17
2	+	−	−	−	−	−	+	−	+	+	+	+	+	−	−	115
3	−	+	−	−	−	+	−	+	−	+	+	+	−	+	−	12
4	+	+	−	−	+	−	−	−	−	+	−	−	+	+	+	5
5	−	−	+	−	+	−	−	+	+	−	+	−	+	+	−	8
6	+	−	+	−	−	+	−	−	+	−	−	+	−	+	+	43
7	−	+	+	−	−	−	+	+	−	−	−	+	+	−	+	50
8	+	+	+	−	+	+	+	−	−	−	+	−	−	−	−	4
9	−	−	−	+	+	+	+	−	−	−	−	+	+	+	−	12
10	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+	7
11	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+	55
12	+	+	−	+	+	−	−	+	+	−	−	+	−	−	−	3
13	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+	42
14	+	−	+	+	−	+	−	+	−	+	−	−	+	−	−	6
15	−	+	+	+	−	−	+	−	+	+	−	−	−	+	−	18
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	1
Main and interaction contribution	− 30	− 102	− 54	− 110	− 214	− 98	+ 50	− 190	+ 122	+ 34	+ 90	+ 158	+ 106	− 186	+ 42	
					↑ Red X			↑ Pink X								